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DYNAMICS OF ELASTIC MEDIA WITH THE DIFFRACTION OF SPHERICAL WAVES

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Consider the distribution of elastic waves in an infinite space with a spherical cavity; These waves are due to the pressure $p(t) = p_0 e^{i\omega t}$, applied to the surface $R = a$.

In the region $R \geq a$, longitudinal spherical waves are appear. The displacement vector u is reduced to one radial component, the displacements depend only on the variables R and t.

The solution of this problem will be sought in the form of a potential *Φ (R, t)* satisfying equation

$$\left(\frac{\partial^2}{\partial R^2} + \frac{2}{R}\frac{\partial}{\partial R} - \frac{1}{c_1^2}\frac{\partial^2}{\partial t^2}\right)\Phi(\mathrm{R,t}) = 0. \qquad (1)$$

The solution of this equation can be represented as a divergent wave

$$\Phi(\mathrm{R,t}) = \frac{A}{R} e^{i\omega\left(t - \frac{R}{c_1}\right)}, \qquad (2)$$

which satisfies the radiation condition as $R \to \infty$. We define the constant A from the boundary condition

$$\sigma_{RR}(a,t) = -p_0 e^{i\omega t}. \qquad (3)$$

such that $\sigma_{RR} = 2\mu \frac{\partial u_R}{\partial R} + \lambda\left(\frac{\partial u_R}{\partial R} + \frac{2u_R}{R}\right) = 2\mu\frac{\partial^2\Phi}{\partial R^2} + \frac{\lambda}{c_1^2}\frac{\partial^2\Phi}{\partial t^2}$,

then, consider the boundary condition (3), we define the constant

$$A = -\frac{p_0 a^3 e^{ai\omega/c_1}}{4\mu\left(1+\frac{ai\omega}{c_1}\right) - (\lambda+2\mu)\left(\frac{ai\omega}{c_1}\right)^2} \qquad (4)$$

The function Φ is also defined by (2). Now it is easy to determine the displacement u_R and other components of the stress state:

$$u_R = \frac{\partial\Phi}{\partial R}, \qquad \sigma_{\vartheta\vartheta} = \sigma_{\varphi\varphi} = \frac{2\mu}{R}\frac{\partial\Phi}{\partial R} + \frac{\lambda}{c_1^2}\frac{\partial^2\Phi}{\partial t^2}. \qquad (5)$$

Therefore, $$u_R = Re\left\{\frac{p_0 a^3 (1+ikR) e^{i(\omega t - k(R-a))}}{R^2(4\mu(1+ika) - (\lambda+2\mu)(ka)^2)}\right\}, \qquad k = \frac{\omega}{c_1} \qquad (6)$$

Parameters of granite: Density: ρ=2.6*103 kg/m3, Lame parameters: λ=2.99*104 MPa, μ= 3.185*104 MPa. Other parameters: Frequency of wave: ω=0.5 sec-1, Radius of cavern: a=15 m. Variables: Initial pressure: p_0. Distance from spherical cavern to ground surface (in the same direction as the radius): R. Time: t

By putting some parameters of granite such as Lame parameters and density, we get some results of displacement and stress at each point in medium.

Figure 1 and Figure 2 are graphs of displacement and stress tensors at initial pressure p_0 = 20 MPa and time t = 0.

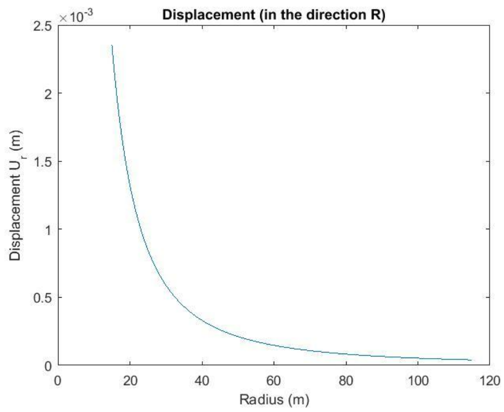

Fig. 1. Graph of displacement depends on R

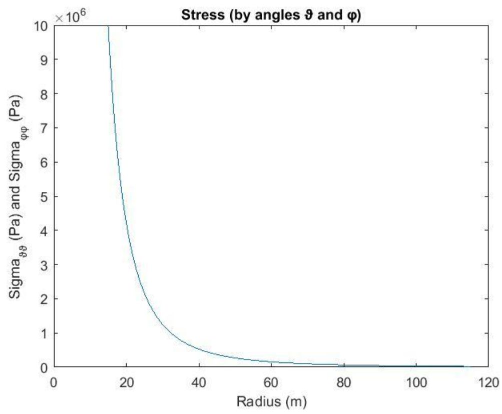

Fig. 2. Changing of stress by angles ϑ and φ

As we see at Figure 2 displacement at t=0 expanentially decline from 2.5mm to approxiamately 0 mm, it means that at distance 115m from center of cavern the displacements of every point in granite rock is insignificantlly.

To sum up, we prove that spherical gas storage cavern with radius 15m, at depth 115m in granite rock, allows to store gas at high pressure as 20 MPa with insignificant deformations of medium. Usually, at a depth of 200 m it is possible to store gas at 2 MPa for aquifers and 4 MPa for salt caverns. This technology is planned to store gas at 15 to 30 MPa. The reasons for these differences are related to the basic principles of the storage concepts. The surrounding rock mass carries the gas pressure load and thus acts as pressure absorber. The rock mass is deformed by the gas pressure load, causing the cavern to expand.

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